

Which A is the best A?

An FFT analysis of Violoncello fingered versus Open Strings

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Introduction

The violoncello (abbreviated hereafter as simply “cello”) has 4 strings, spaced at an interval of a perfect fifth apart. The strings are, in descending order, A, D, G, and C. The A string sounds at 220 Hz (compare with the violin, whose A string sounds at an octave higher, at 440 Hz).

Table 1: frequencies at which Cello strings sound (ignoring even vs. tempered playing, and the tendency to tune orchestras to 441 or even 442 Hz as opposed to 440 Hz).

| A String (Hz) | D String (Hz) | G String (Hz) | C String (Hz) |
|---------------|---------------|---------------|---------------|
| 220 | 146.8 | 98 | 65.4 |

The Harmonic Series in Graphical format

The fact that the cello A sounds at 220 Hz does not mean that that is its only frequency, and that we can represent the cello sound as a simple sine wave $x(t) = A\sin((220 \cdot 2\pi)t)$. What you’ll find is that that sine wave sounds like a pure, piercing tone to our ears. Because of the cello’s shape and other physical attributes such as its material, the way the player is playing it, and other factors, there are several frequencies that show up “within” the notes, called “overtones.” The frequencies that are encapsulated within the notes that are sounded peak at integer multiples of the frequency.

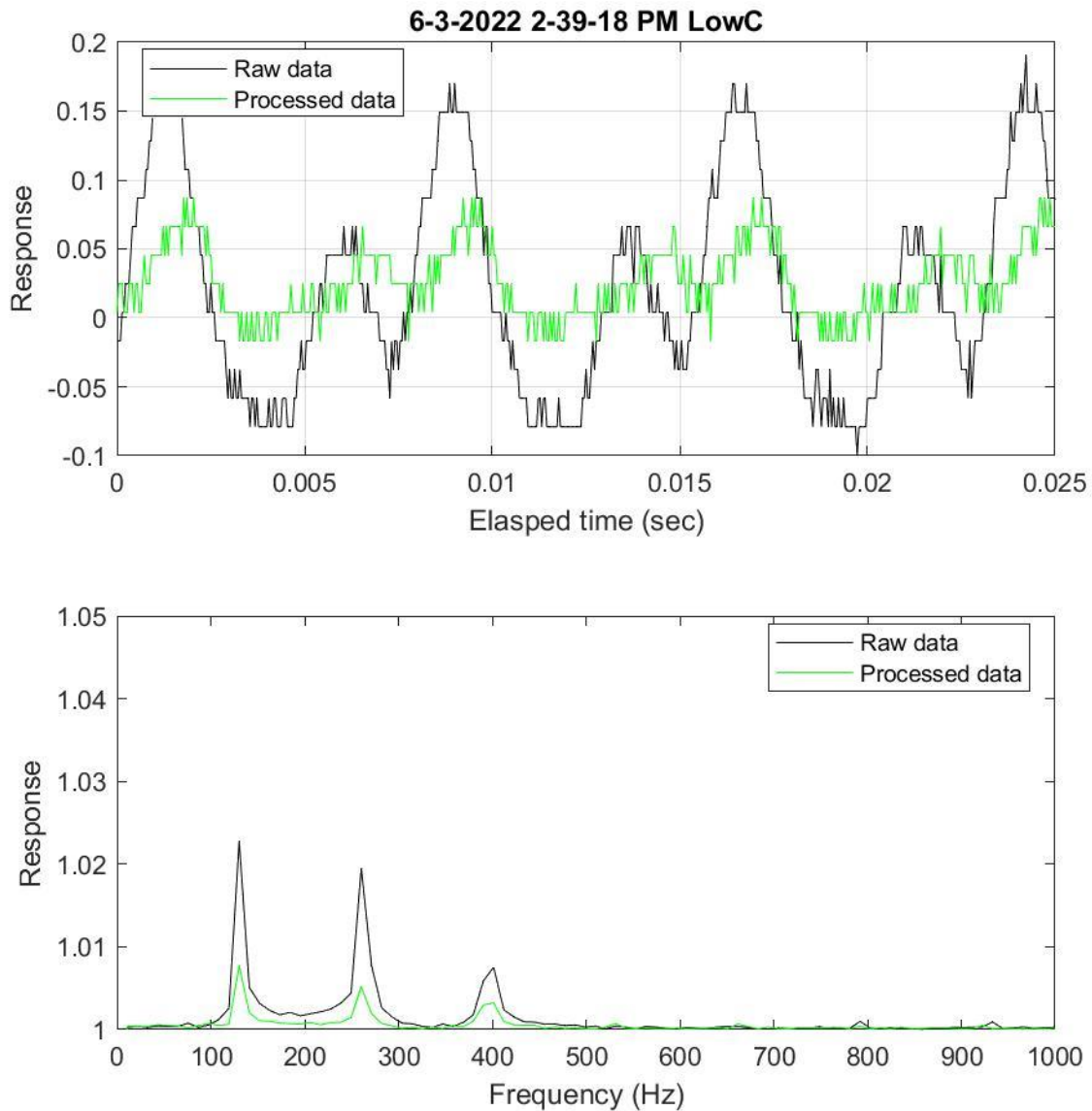


Figure 1: A sample FFT response of a guitar playing at its Lowest C (130.8 Hz)

Figure 1 shows a guitar playing one octave above the cello's lowest note of C at 64.5 Hz. The first and highest frequency peak occurs at 130.8 Hz, and that is the note that is heard. However, there are two more clear peaks on top of that. The harmonic series tells us that the second peak should occur at around 261.6 Hz, which is clearly what is happening in Figure 1. The third peak is also in the harmonic series, but it is a fifth above the previous peak instead of an octave. Further explanation of the harmonic series is below:

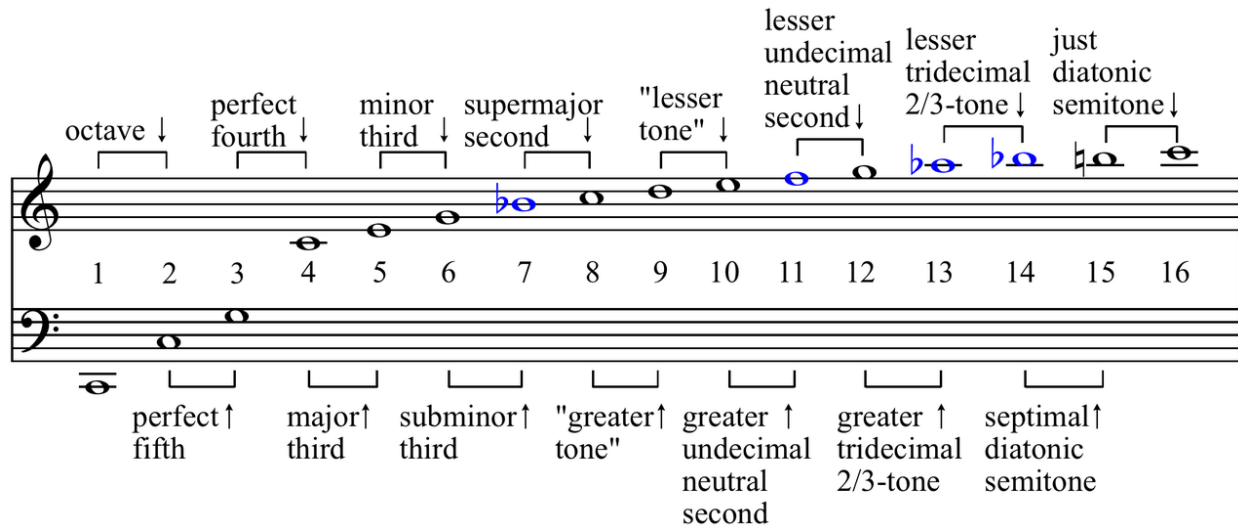


Figure 2: The harmonic series of low C (64.5 Hz) in musical notation

Figure 2 shows the notes that are in the overtone or harmonic series of low C, which is the cello's lowest string. A frequency response analysis of the cello playing a C should result in peaks at the frequencies associated with some, or all these notes, with the highest peak at the frequency of 64.5 Hz.

In order, the peaks should appear at the frequencies in Table 2.

Table 2: A list of the first 5 frequencies in the harmonic series of C 64.4 Hz.

| Note | Frequency (Hz) |
|--------------------------|----------------|
| C (lowest) | 64.41 |
| C (one octave higher) | 130.81 |
| G (one fifth higher) | 196.00 |
| C (one fourth higher) | 261.63 |
| E (a major third higher) | 329.63 |
| G (a minor third higher) | 392.00 |

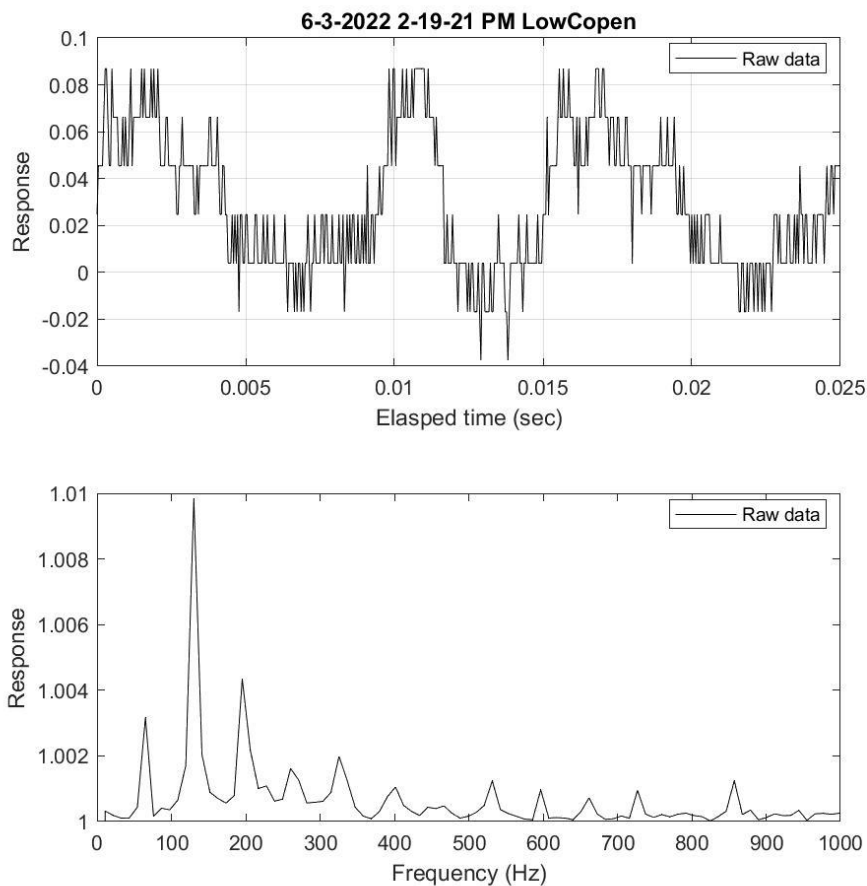


Figure 3: A sample of the FFT response to a cello playing

Figure 3 shows the captured FFT that has peaks at all the harmonic series described by Table 2. Something unexpected is that the highest frequency peak is at the first harmonic (the second peak) instead of at the fundamental frequency. Since I, the author of this analysis, played the note during data capture, I can attest that the note did sound like a low C at 64.4 Hz, yet the amplitude of the first harmonic appears almost three times higher. One explanation I can give is that when playing the C string, all other strings on the instrument were uncovered, and so the instrument was allowed to resonate fully. Perhaps the natural resonance of the instrument is much stronger at 130.81 Hz.

The other explanation may be the data collection style itself. The mic had difficulty picking up any sound from the cello even when I was playing quite loudly, and so it's safe to say the mic was not in good working condition. It may just have been more sensitive to a range of frequencies above 100 Hz or so. It's difficult to say because there is no comparison of low C's with the guitar, which had a different method of data collection, as the guitar's lowest note is still higher than the cello's.

Fingered vs. Open Strings

Now that we have established what the frequency peaks are, let's talk about fingered vs. unfingered strings and analyze any possible difference in frequency response.

Since there are four strings on the cello, there are four different options of how to play the same note, with a bottom limit at its highest string, A. In order to create different pitches on a stringed instrument, players either change the tension of the strings, which is very difficult to do quickly and with accuracy, or they shorten the string using their fingers. By using 4 open strings tuned at different pitches, the cellist has four different "bottom starting positions," from which to use their fingers to change the notes. The strings can be shortened by intervals that far exceed the intervals between strings, hence, the same note can be played on multiple different strings.

Each string has a vastly different tone quality, with the A string presenting as the brightest string, the D string with a more muted tone, the G and C string very rich and complex. The tone quality of instruments is called "timbre" and can vary between player and instrument. Using the same instrument, I tried to find out if the different tone quality of the strings could be seen through a Fourier analysis.

Figures 4 through 7 show the FFT response of the cello playing the note A 220 Hz on each string, including the A string (no fingers down).

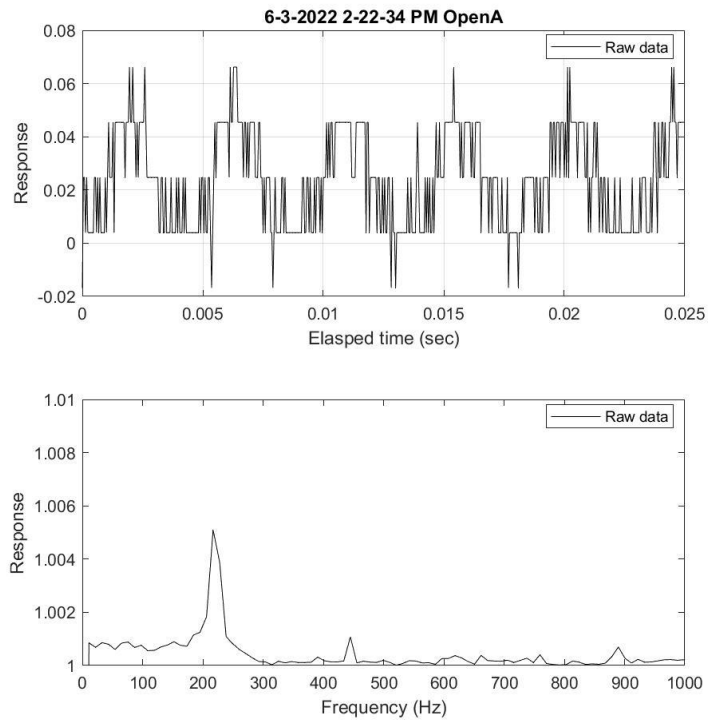


Figure 4: Frequency response of a cello playing A 220 Hz on the Open A string

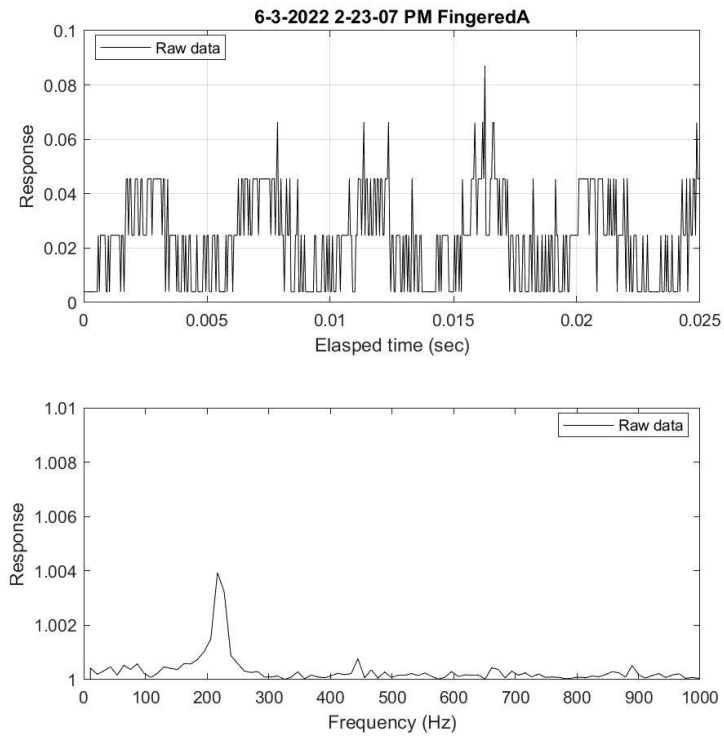


Figure 5: Frequency response of a cello playing A 220 Hz on the D string

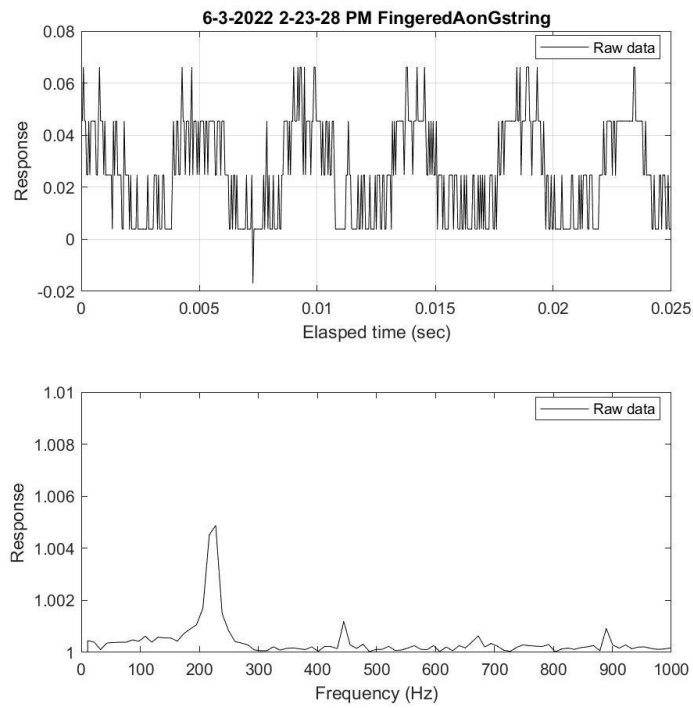


Figure 6: Frequency response of a cello playing A 220 Hz on the G string

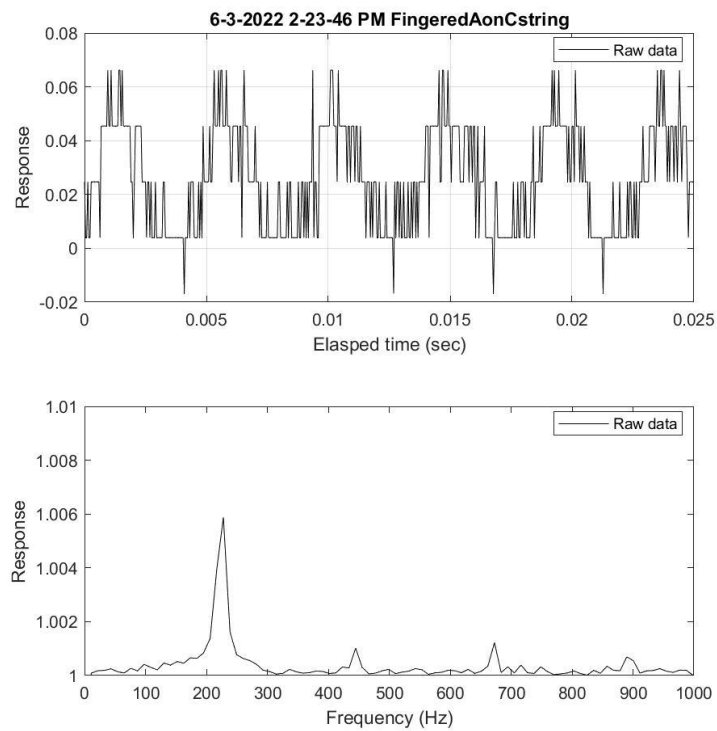


Figure 7: Frequency response of a cello playing A 220 Hz on the C string

Immediately I can tell that the graphs are not all that different. The fundamental frequency of A 220 Hz is the first and most prominent peak in all 4 charts. The next two to three peaks should be at A 440 Hz (an octave higher), E 659.25 Hz (a fifth above that), and A 880 Hz (another fifth above that).

The string with the highest amplitude at 220 Hz is the C string (Figure 7), which surprises me. Perhaps the “loud” resonance at A 220 Hz is due to the C string having the biggest mass of all the other strings. Second highest is the A string (Figure 4), which resonates very well at A 220 Hz, most likely because that is its fundamental frequency, undamped by a human finger.

The string with the least-defined peaks at the harmonic series is the D string (Figure 5). The fundamental frequency is clearly visible, but the other peaks at A 440, E 660, and A 880 are almost indistinguishable from the other overtones. The D string is somewhat muted and darker in tone quality in comparison to the rest of the instrument, and this difficulty in establishing a clear harmonic series from the FFT graph might be a result of that. However, the D string is also the quietest string on the instrument, and given that the mic was having difficulty picking up the sound of the cello, that may be another explanation for the result in Figure 5.

Interesting as well is the C string’s large peak at E 660 Hz, which is the second harmonic in the series. Compared with A played on the other strings, the A on the C string has the sharpest and highest peak at E 660 Hz. Perhaps this contributes to the somewhat “hairy” sound of playing A on the C string – the note has a breathy quality to it, almost like you can’t quite tell which note is playing.

Conclusion

The peaks in the FFT graphs analyzed here all line up exactly with expected values from the harmonic series. In addressing the difference in tone qualities of A 220 Hz played on different strings on the cello, it is difficult to say whether the string had that much of an effect and why there was an implied effect at all given the low level of fidelity of the recording equipment. However, the slight differences there were implied there may be a connection between the varying tone quality of individual cello strings and an analysis of its FFT frequency response. Further, more targeted experiments and data collection methods would be necessary for a more thorough analysis.

Resources

<http://hyperphysics.phy-astr.gsu.edu/hbase/Music/cello.html>

[https://en.wikipedia.org/wiki/Harmonic_series_\(music\)](https://en.wikipedia.org/wiki/Harmonic_series_(music))

<https://mixbutton.com/mixing-articles/music-note-to-frequency-chart/>

Appendix I

```
close all
clearvars
clc
```

%Guitar Plots

```
PARENT_DIR_GTR = './Guitar/';
THESE_DATAFILES_GTR = dir([PARENT_DIR_GTR '*.txt'])

for i = 1:length(THESE_DATAFILES_GTR)
    [etimeSec, rawData, processedData] = importGuitarData([PARENT_DIR_GTR
THESE_DATAFILES_GTR(i).name])
    [freqHzRaw, fftRaw] = computeFFT(etimeSec, rawData);
    [freqHzProcessed, fftProcessed] = computeFFT(etimeSec, processedData);

    outFilename = strrep(THESE_DATAFILES_GTR(i).name, '.txt', '');
    chartTitle = strrep(outFilename, '_', ' ');

    close all
    figure('Color','w','Units','inches','Position',[1,1,6.5,6.5])

    subplot(2,1,1)
    plot(etimeSec, rawData, 'k')
    hold on
    plot(etimeSec, processedData, 'g')
    grid on
    xlim([0 0.025])
    xlabel('Elasped time (sec)')
    ylabel(['Response'])
    legend('Raw data', 'Processed data', 'Location', 'Best')
    title(chartTitle)

    subplot(2,1,2)
    plot(freqHzRaw, fftRaw, 'k')
    hold on
    plot(freqHzProcessed, fftProcessed, 'g')
    xlim([0 1000])
    ylim([1 1.05])
    xlabel('Frequency (Hz)')
    ylabel(['Response'])
    legend('Raw data', 'Processed data', 'Location', 'Best')
    saveas(gcf, ['./' outFilename '.jpg'], 'jpg')
    close all
end
```

%% Section II

```
close all
clearvars
clc
```

%Cello

```

PARENT_DIR_VLC = './Cello/';
THESE_DATAFILES_VLC = dir([PARENT_DIR_VLC '*.txt'])

for i = 1:length(THESE_DATAFILES_VLC)
    [etimeSec, rawData] = importCelloData([PARENT_DIR_VLC
THESE_DATAFILES_VLC(i).name])
    [freqHzRaw, fftRaw] = computeFFT(etimeSec, rawData);

    outFilename = strrep(THESE_DATAFILES_VLC(i).name, '.txt', '');
    chartTitle = strrep(outFilename, '_', ' ');

    close all
    figure('Color','w','Units','inches','Position',[1,1,6.5,6.5])

    subplot(2,1,1)
    plot(etimeSec, rawData, 'k')
    hold on
    grid on
    xlim([0 0.025])
    xlabel('Elapsed time (sec)')
    ylabel(['Response'])
    legend('Raw data', 'Processed data', 'Location', 'Best')
    title(chartTitle)

    subplot(2,1,2)
    plot(freqHzRaw, fftRaw, 'k')
    hold on
    xlim([0 1000])
    ylim([1 1.01])
    xlabel('Frequency (Hz)')
    ylabel(['Response'])
    legend('Raw data', 'Processed data', 'Location', 'Best')
    saveas(gcf, ['./' outFilename '.jpg'], 'jpg')
    close all

end

```

Appendix II

```
function [freqHz, Y] = computeFFT(etimeSec, response)

etimeSec = etimeSec-etimeSec(1);
response = response-response(1);
sampleRateHz = 1/(etimeSec(2) - etimeSec(1));

L = length(etimeSec);
nFFT = 2^nextpow2(L);
Y = fft(response, nFFT);
Y = abs(Y/nFFT);
Y = Y(1:nFFT/2+1);
Y(2:end-1) = 2.*Y(2:end-1);
freqHz = sampleRateHz*(0:(nFFT/2))/nFFT;
end
```